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Stability and mass of point particles

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Abstract

In this paper we consider classical point particles in full interaction with an arbitrary number of dynamical scalar and (abelian) vector fields. It is shown that the requirement of stability —vanishing self-force— is sufficient to remove the well-known inconsistencies of the classical theory: the divergent self-energy, as well as the failure of Lorentz-covariance of the energy-momentum when including the contributions of the fields. As a result, in these models the mass of a point particle becomes finitely computable. We discuss how these models are connected to quantum field theory via the path-integral representation of the propagator.

1 Introduction

The origin of particle masses is one of the recurrent themes of discussion in fundamental physics. The present consensus is that the masses of all known particles have a field-theoretical explanation: quark, lepton and vector boson masses are supposed to have their origin in the vacuum expectation value of a scalar field [1, 2]. The account of the rest energy of particles is completed by including a contribution from the Coulomb-, Yukawa- and other static fields coupling to the particle.

In the standard model, and also in the simpler case of classical and quantum electrodynamics, the contributions of these fields to the masses of particles are not computable: they are infinite, and infinite compensating terms have to be included in the calculations to get finite results for the values of the physical observables. These compensating terms are usually attributed to the effect of unknown physics at smaller distance scales. Thus particle masses can be accommodated in field theory, but the question whether they have a fully field-theoretical explanation remains open: ultimately the explanation of the particle spectrum is presumably to be found in Planck-scale physics; indeed, a truly finite theory of quantum gravity, e.g. string theory, *should* allow the computation of the mass spectrum of all particle states. Even so, in such a theory the masses of known particles, far below the Planck scale, might well have a completely field-theoretical ('low-energy') explanation.

In this paper I explore anew the possibility of a purely field-theoretical explanation for (at least some) particle masses. I construct a class of fully interacting particle-field models in which the classical mass is finite and fully computable in terms of the self-fields of the charges carried by the particle. I show how mass generation (including the equivalent of the Brout-Engert-Higgs effect) can be incorporated in classical particle dynamics for the case of a particle coupled to N_v vector fields, with vector charges q_α and mass μ_α ($\alpha = 1, ..., N_v$), and to N_s scalar fields, with scalar charges g_i, mass μ_i and vacuum expectation values f_i ($i = 1, ..., N_s$). More specifically, the following expression can be derived for the total particle mass, in natural units ($c = \hbar = 1$):

$$M = m + \sum_i g_i f_i + \frac{1}{8\pi} \left(\sum_i g_i^2 \mu_i - \sum_\alpha q_\alpha^2 \mu_\alpha \right), \tag{1}$$

where m is any contribution of non-field theoretical origin; if $m = 0$ the total mass M is determined purely by the fields. A slightly less general form of this result (without the scalar vacuum expectation values) has actually been derived in the early days of quantum field theory [3], but here I give a fully classical account: I show that the finite result hinges on the classical particle being stable and not subject to self-acceleration, thereby implying full covariance of the energy-momentum of the particle-field system. Thus all inconsistencies of the classical

theory of charged particles[1] are removed.

The relation between this result and quantum field theory is also discussed. An improved version of perturbation theory is outlined, which might preserve some of the desirable properties of the classical model, in particular in combination with supersymmetry.

This paper is structured as follows. In sect. 2, two definitions of mass are recalled; it is shown how to compute them in the almost trivial case of a free particle. In sect. 3, I present a class of models of particles interacting with an arbitrary number of scalar and (abelian) vector fields. The equations of motion for a single particle are solved simultaneously with the field equations, taking full account of the back reaction of the particle acting as a source for the fields. It is shown that the requirement of stability implies two relations between the coupling constants and the ranges of the fields. In sect. 4, the energy-momentum tensor of the particle and its fields is computed, and it is shown that the stability condition implies both finiteness of the total mass and covariance of the total energy-momentum. In sect. 5 the mass is computed by the Hamilton-Jacobi method, giving the same result, eq.(1). In sect. 6, the connection with quantum field theory is made using the path-integral formalism for the (full) propagator of the corresponding model. In sect. 7, I discuss the results and draw some conclusions.

2 Mass

The equivalence principle equates the inertial an gravitational mass of point particles. In a special relativistic context, the inertial mass is defined by the kinematics, i.e. the dispersion relation between energy and momentum:

$$p_\mu^2 + m^2 c^2 = -\frac{E^2}{c^2} + \vec{p}^{\,2} + m^2 c^2 = 0. \tag{2}$$

The gravitational mass is defined by the energy-momentum tensor of the particle, acting as the source for gravitational fields in the Einstein equations. For a direct comparison with (2), we should also consider it in the special relativistic limit of flat Minkowski space. In this limit it is a symmetric, divergence-free tensor field $T_{\mu\nu}(x)$: $\partial_\mu T^{\mu\nu} = 0$, with the property that, for the space-like 3-dimensional hypersurface Σ: $x^0 =$ constant, the conserved four-momentum of eq.(2) is

$$p^\mu = \frac{1}{c} \int_\Sigma d^3x \, T^{\mu 0}. \tag{3}$$

In particular, in the rest frame ($\vec{p} = 0$)

$$mc^2 = \int_\Sigma d^3x \, T^{00}, \tag{4}$$

[1]For a modern discussion see for instance ref.[4].

provided the integral on the r.h.s. of eq.(3) is well-defined, transforming as a contravariant four-vector under Lorentz transformations.

As an illustration, and as a preparation for the more complicated models to be considered later, I first discuss the case of the free point mass, described by the action [5]

$$S_0 \,=\, -\,mc^2 \int d\lambda \sqrt{-\left(\frac{1}{c} \frac{d\xi^\mu}{d\lambda} \right)^2}. \tag{5}$$

Here $\xi^\mu(\lambda)$ are the co-ordinates of the particle as a function of the worldline-parameter λ. Note that the action is actually reparametrization invariant; a natural and common choice for λ is to equate it to proper time:

$$d\lambda \,=\, d\tau \,\equiv\, \frac{1}{c} \sqrt{-d\xi_\mu^2}. \tag{6}$$

The canonical momentum conjugate to ξ^μ is

$$p^\mu \,=\, mu^\mu \,=\, m \frac{d\xi^\mu}{d\tau}. \tag{7}$$

By definition of τ it satisfies the mass-shell condition (2). The momentum can also be obtained from a divergence-free energy-momentum tensor as in eq.(3), by taking

$$\begin{aligned} T^{\mu\nu}(x) \;&=\; mc \int d\tau \, \frac{d\xi^\mu}{d\tau} \frac{d\xi^\nu}{d\tau} \, \delta^4 \left(x - \xi(\tau) \right) \\ &=\; \left[m \, \frac{d\xi^\mu}{d\tau} \frac{d\xi^\nu}{d\tau} \, \delta^3 \left(\frac{\vec{x} - \vec{\xi}(t)}{\sqrt{1 - \vec{v}^2/c^2}} \right) \right]_{\xi^0 = ct}. \end{aligned} \tag{8}$$

Obviously, in the rest frame $d\xi^0 = c\, d\tau$ and eq.(4) is satisfied.

An alternative to this scheme is provided by the Hamilton-Jacobi method. The conservation of four-momentum for a free particle allows us to write

$$p^\mu \,=\, m \, \frac{\xi_f^\mu - \xi_i^\mu}{\tau_f - \tau_i}, \tag{9}$$

for motion during a fixed proper-time interval (τ_i, τ_f), with (ξ_i^μ, ξ_f^μ) representing the initial and final co-ordinates of the corresponding stretch of worldline. Inserting the solution of the equation of motion back into the action gives

$$S_0^{cl} \,=\, -mc \sqrt{-\left(\xi_f^\mu - \xi_i^\mu \right)^2} \,=\, -mc^2 \left(\tau_f - \tau_i \right). \tag{10}$$

From this expression the Hamilton-Jacobi equation

$$p_\mu(\tau_f) = \frac{\partial S_0^{cl}}{\partial \xi_f^\mu}, \tag{11}$$

can be verified directly. Thus we observe, that the constant in front of the proper-time interval in the classical action defines the mass.

One of the main results obtained below is, that for particles interacting with scalar and vector fields in a consistent way the one-particle Hamilton-Jacobi function is precisely of the form (10), with a renormalized value of the mass parameter. This renormalized value then represents the physical mass, as is verified independently from a calculation of the stress-energy tensor.

3 Particles in interaction with dynamical fields

In this section we extend the previous analysis to models of a relativistic particles interacting with N_s scalar fields φ_i $(i = 1, ..., N_s)$, and N_v vector fields A_μ^α $(\alpha = 1, ..., N_v)$. We take these fields to be fully dynamical, with (a priori arbitrary) ranges $\lambda_{i,\alpha} = \mu_{i,\alpha}^{-1}$, whilst the scalar fields can also have a vacuum expectation value $\langle \varphi_i \rangle = f_i$. We do not consider self-interactions of these fields, so our vector fields are taken to be of abelian type. Non-abelian interactions would require the introduction of more than one type of particle. Thus our model could apply to a simplified version of the electroweak standard model based on $U(1) \times U(1)$, in which a (scalar) electron couples to the photon and the Z^0, but not to charged vector bosons $W^\pm$.

With these assumptions we introduce a particle model based on the following action

$$\begin{aligned} S_{field} &= \int d^4x \left\{ - \sum_{i=1}^{N_s} \left[\frac{1}{2} \left(\partial_\mu \varphi_i \right)^2 + \frac{\mu_i^2}{2} \left(\varphi_i - f_i \right)^2 + \rho_i \, \varphi_i \right] \right. \\ & \left. - \sum_{\alpha=1}^{N_v} \left[\frac{1}{4} \left(F_{\mu\nu}^\alpha \right)^2 + \frac{\mu_\alpha^2}{2} \left(A_\mu^\alpha \right)^2 - \frac{1}{c} A_\mu^\alpha j_\alpha^\mu \right] \right\}, \end{aligned} \tag{12}$$

where the scalar charge densities ρ_i and vector current densities j_α^μ are defined by

$$\begin{aligned} \rho_i(x) &= g_i \int d\lambda \sqrt{- \left(\frac{d\xi^\mu}{d\lambda} \right)^2} \, \delta^4 \left(x - \xi(\lambda) \right) \\ &= g_i \, \delta^3 \left(\frac{\vec{x} - \vec{\xi}(t)}{\sqrt{1 - \vec{v}^2/c^2}} \right), \end{aligned}$$

$$
\begin{aligned}
j^\mu_\alpha(x) &= q_\alpha c \int d\lambda \, \frac{d\xi^\mu}{d\lambda} \, \delta^4 \left(x - \xi(\lambda) \right) \\
&= q_\alpha \, u^\mu \, \delta^3 \left(\frac{\vec{x} - \vec{\xi}(t)}{\sqrt{1 - \vec{v}^2/c^2}} \right),
\end{aligned}
\tag{13}
$$

where u^μ is the four-velocity. Note that the coupling of the scalar fields to the scalar charge density represents a kinetic term for the particle of Einstein-type, with a space-time dependent mass $\sum g_i \varphi_i(x)$. It is of course possible to add a separate kinetic term of the type S_0, as in eq.(5), involving a strictly mechanical mass. However, one can derive the above models from a quantum field theory through the path-integral representation of the propagator, as for example in [6]-[9]; in that case the additional kinetic term is absent.

In order to compute the contributions of the fields to the mechanical properties of the particle, we first consider the field equations

$$
\begin{aligned}
\left(-\Box + \mu_i^2 \right) \left(\varphi_i - f_i \right) &= -\rho_i, \\
\left[\left(-\Box + \mu_\alpha^2 \right) \eta^{\mu\nu} + \partial^\mu \partial^\nu \right] \, A^\alpha_\nu &= \frac{1}{c} \, j^\mu_\alpha.
\end{aligned}
\tag{14}
$$

Any solution of these equations consists of a particular solution of the inhomogeneous Klein-Gordon or Proca equation, for which we take the retarded Green's function, plus a solution of the homogeneous equation. In the case of a particle moving with constant velocity, the retarded Green's functions simplify to take the form of the usual Coulomb-Yukawa potentials appropriately boosted to a moving frame:

$$
\begin{aligned}
\varphi_i(x) &= \varphi_i^{free} + f_i - \frac{g_i}{4\pi} \, \frac{e^{-\mu_i R_{ret}}}{R_{ret}}, \\
A^\alpha_\mu &= A^{\alpha \, free}_\mu + u_\mu \, \frac{q_\alpha}{4\pi c} \, \frac{e^{-\mu_\alpha R_{ret}}}{R_{ret}}.
\end{aligned}
\tag{15}
$$

Here the retarded distance parameter $R_{ret} = |\vec{R}_{ret}|$ is obtained by boosting the relative position vector $\vec{r} = \vec{x} - \vec{\xi}$ in the lab frame back to the rest frame. Hence we get

$$
R_i^{ret} = \left(\delta_{ij} - \frac{v_i v_j}{\vec{v}^2} \right) \, r_j + \frac{v_i}{\sqrt{1 - \vec{v}^2/c^2}} \left(\frac{\vec{v} \cdot \vec{r}}{\vec{v}^2} - t \right). \tag{16}
$$

For example, if the particle sits in the origin of its rest frame, which moves with velocity v in the direction of the z-axis of the lab system, this reduces to

$$\vec{R}_{ret} \;=\; \left(x, y, \frac{z - vt}{\sqrt{1 - v^2/c^2}} \right), \tag{17}$$

and therefore

$$R_{ret} \;=\; \sqrt{x^2 + y^2 + \frac{(z - vt)^2}{1 - v^2/c^2}}, \tag{18}$$

with (ct, x, y, z) the co-ordinates in the lab frame. Note also, that the solution of the inhomogeneous Klein-Gordon equation is shifted by the constant f_i. In line with standard terminology we refer to the solutions $(\varphi_i^{free}, A_\mu^{\alpha\, free})$ of the homogeneous equations as the *radiation fields*, the particular solution of the inhomogeneous equation taking the form of the *Coulomb* and *Yukawa field* in the vector and scalar case, respectively. The static fields always accompany the particle and contribute to its inertial and gravitational mass.

Next we turn to the equation of motion of the particle. Varying ξ^μ, and allowing for a additional mechanical mass term, the total action is stationary if

$$\frac{1}{c^2}\frac{d}{d\tau}\left[\left(mc^2 + \sum_i g_i \varphi_i(\xi) \right) \frac{d\xi^\mu}{d\tau} \right] \;=\; -\sum_i g_i \partial^\mu \varphi_i(\xi) \;+\; \sum_\alpha q_\alpha F^{\mu}_{\alpha\,\nu}(\xi) \frac{d\xi^\nu}{d\tau}. \tag{19}$$

Now we require that in the absence of external fields the free particle, dressed with its Coulomb-Yukawa fields, is at rest or moves at constant velocity: it should not exert a net force on itself and the acceleration must vanish. Then

$$\frac{d^2 \xi^\mu}{d\tau^2} \;=\; 0, \tag{20}$$

with the result that

$$\sum_i g_i \partial_\nu \varphi_i(\xi)\, \frac{1}{c^2}\frac{d\xi^\nu}{d\tau}\frac{d\xi^\mu}{d\tau} \;=\; -\sum_i g_i \partial^\mu \varphi_i(\xi) \;+\; \sum_\alpha q_\alpha F^{\mu}_{\alpha\,\nu}(\xi) \frac{d\xi^\nu}{d\tau}. \tag{21}$$

In the rest frame, in which all fields are static, this condition reduces to

$$-\sum_i g_i \vec{\nabla} \varphi_i(\xi) \;+\; \sum_\alpha q_\alpha \vec{E}_\alpha(\xi) \;=\; 0, \tag{22}$$

where $\vec{E}_\alpha$ denote the electric components of the field strength tensor $F^\alpha_{\mu\nu}$, and ξ is the position of the particle, which in the rest frame is actually the origin, according to our conventions. Of course, each term in eq.(22) is singular by itself, as follows from the explicit expressions for the fields in eq.(15) upon putting the free radiation fields equal to zero. However, the singular parts may now cancel between the scalar and vector fields, making the full sum of terms vanish. Explicitly, eq.(22) for the fields in the rest frame becomes

$$\lim_{R\to 0} \vec{\nabla} \left(- \sum_i g_i^2 \frac{e^{-\mu_i R}}{R} + \sum_\alpha q_\alpha^2 \frac{e^{-\mu_\alpha R}}{R} \right) = 0. \tag{23}$$

The left-hand side is a Laurent series in R with a second order pole and a constant term, all other terms vanishing as $R \to 0$. The residue of the $1/R^2$-term, and the constant term in the expansion are removed if and only if the following two conditions are satisfied:
(A) for the infinite part

$$\sum_i g_i^2 = \sum_\alpha q_\alpha^2; \tag{24}$$

(B) for the finite part

$$\sum_i g_i^2 \mu_i^2 = \sum_\alpha q_\alpha^2 \mu_\alpha^2. \tag{25}$$

Therefore in these models the following observations hold:
– stability condition (A) requires both vector and scalar fields to be present;
– if all vector fields are massless, condition (B) requires all scalar fields should be massless as well;
– conversely, if one or more scalar fields have a non-zero mass, (B) implies that the particle must couple to at least one *massive* vector field (and vice versa); for example, if our scheme would apply to some kind of neutrino's, the coupling of the neutrino to the Z^0 would suggest that neutrino's couple also to the Higgs fields and thus have a mass.

We conclude, that we have found a consistent, finite solution to the *complete* system of classical dynamical equations for the particle and the fields, including back reaction; consistency of this solution requires relations between the coupling constants and masses of the fields of the form (24) and (25).

4 The stress-energy tensor

The stress-energy tensor of the system of particle and fields in general admits the following decomposition:

$$T_{\mu\nu} = T_{\mu\nu}^{particle} + T_{\mu\nu}^{scalar} + T_{\mu\nu}^{vector} + \Lambda \eta_{\mu\nu}, \tag{26}$$

where the various terms refer to the contribution of the particle, the scalar fields and the vector fields, and Λ is an arbitrary constant, which is automatically conserved and hence in principle allowed.

The stress-energy tensor is a symmetric real matrix and therefore can be decomposed in terms of a pseudo-orthonormal set of eigenvectors $n_{(\lambda)}$, $\lambda = 0, 1, 2, 3$, with eigenvalues $\alpha_{(\lambda)}$ which in general are functions of the space-time point and the position of the particle:

$$T^{\mu}_{\ \nu}\, n^{\nu}_{(\lambda)} \;=\; \alpha_{(\lambda)}\, n^{\mu}_{(\lambda)}, \qquad\qquad \eta_{\mu\nu}\, n^{\mu}_{(\lambda)} n^{\nu}_{(\lambda')} \;=\; \eta_{\lambda\lambda'}. \tag{27}$$

In our model the eigenvectors are determined completely by the geometry, to wit the spherical symmetry in the rest frame of the particle and the Lorentz boost to the lab frame; therefore the eigenvectors are actually the same for the various contributions to $T_{\mu\nu}$ listed above. For a particle moving with velocity v in the z-direction, this universal basis has the form

$$\begin{aligned}
n^{\mu}_{(0)} &= \left(\frac{1}{\sqrt{1-v^2/c^2}}, 0, 0, \frac{1}{\sqrt{1-v^2/c^2}}\, \frac{v}{c} \right), \\
n^{\mu}_{(1)} &= \left(\frac{(z-vt)}{R(1-v^2/c^2)}\, \frac{v}{c}, \frac{x}{R}, \frac{y}{R}, \frac{(z-vt)}{R(1-v^2/c^2)} \right), \\
n^{\mu}_{(2)} &= \left(\frac{-\sqrt{x^2+y^2}}{R\sqrt{1-v^2/c^2}}\, \frac{v}{c}, \frac{x}{\sqrt{x^2+y^2}}\, \frac{(z-vt)}{R\sqrt{1-v^2/c^2}}, \right. \\
&\qquad\qquad \left. \frac{y}{\sqrt{x^2+y^2}}\, \frac{(z-vt)}{R\sqrt{1-v^2/c^2}}, \frac{-\sqrt{x^2+y^2}}{R\sqrt{1-v^2/c^2}} \right), \\
n^{\mu}_{(3)} &= \left(0, \frac{-y}{\sqrt{x^2+y^2}}, \frac{x}{\sqrt{x^2+y^2}}, 0 \right).
\end{aligned} \tag{28}$$

In these equations $R = R_{ret}$, given by (18). In the rest frame the expressions simplify considerably and can be written in spherical co-ordinates as

$$\begin{aligned}
n_{(0)} &= (1, 0, 0, 0)\,, \\
n_{(1)} &= (0, \sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)\,, \\
n_{(2)} &= (0, \cos\theta\cos\varphi, \cos\theta\sin\varphi, -\sin\theta)\,, \\
n_{(3)} &= (0, -\sin\varphi, \cos\varphi, 0)\,.
\end{aligned} \tag{29}$$

We can now decompose the stress-energy tensor in terms of this basis as follows

$$T_{\mu\nu} = \sum_{\lambda} \alpha_{(\lambda)}\, n_{(\lambda)\mu} n_{(\lambda)\nu}. \tag{30}$$

where the eigenvalues $\alpha_{(\lambda)}$ are Lorentz invariant. Next we observe, that the time-like eigenvector is the normalized four-velocity: $n^{\mu}_{(0)} = u^{\mu}/c$. Therefore a consistent one-particle theory should yield

$$p^\mu = \frac{1}{c}\int_\Sigma d^3x\, T^{\mu 0} = Mc\, n^\mu_{(0)}, \tag{31}$$

where the constant M represents the physical mass of the particle, made up from contributions of all terms in eq.(26):

$$Mc^2 = cp\cdot n_{(0)} = \int_\Sigma d^3x\, \alpha_{(0)}\, n^0_{(0)}. \tag{32}$$

In particular, in the rest frame

$$Mc^2 = \int_\Sigma d^3x\, \alpha_{(0)}. \tag{33}$$

To obtain the results (31)–(33) we require that the integrals over the stress components $(\alpha_{(1)}, \alpha_{(2)}, \alpha_{(3)})$ in the decomposition (30) of $T_{\mu\nu}$ vanish. It turns out that this is guaranteed if condition (A), eq.(24), for the coupling constants is satisfied. In particular, this condition gets rid of the factors 4/3 which appear in the original computation of the ratio between electromagnetic and kinematic mass because of the Poincaré stresses in the classical electron theory [4]. As a result, we can compute the physical mass M directly in the rest frame, where the calculation is rather simple.

A remarkable result is, that from the same condition (A) it follows, that the physical mass M is finite. This is surprising, because the energy contained in the Coulomb- and Yukawa-fields is infinite, and in this case they add up rather than subtract. What saves the model is, that the interaction of the particle with its own scalar field gives an equally singular negative contribution, cancelling the diverging contribution of the pure field term. Physically this can be understood from the attractive character of scalar forces.

We now demonstrate these results by an explicit computation. The contribution of the particle to the stress-energy tensor is

$$\begin{aligned} T^{particle}_{\mu\nu} &= \frac{1}{c}\int d\tau \left(mc^2 + \sum_i g_i \varphi_i(\xi)\right) \frac{d\xi_\mu}{d\tau}\frac{d\xi_\nu}{d\tau}\, \delta^4\left(x - \xi(\tau)\right) \\ &= \left(mc^2 + \sum_i g_i \varphi_i(\xi)\right) \delta^3\left(\frac{\vec{x} - \vec{\xi}(t)}{\sqrt{1 - v^2/c^2}}\right) n_{(0)\mu} n_{(0)\nu}. \end{aligned} \tag{34}$$

Thus the only non-zero eigenvalue of the particle term in the stress-energy tensor is $\alpha_{(0)}$, which in the rest frame becomes the T_{00} component. From eq.(15) we then obtain the rather singular explicit expression

$$\alpha^{part}_{(0)} = \left(mc^2 + \sum_i g_i f_i - \sum_i g_i^2 \frac{e^{-\mu_i R}}{4\pi R}\right) \delta^3\left(\vec{R}\right). \tag{35}$$

Next we consider the scalar fields. The contribution of the scalar fields to the stress-energy tensor takes the form

$$T_{\mu\nu}^{scalar} = \sum_i \left(\partial_\mu\varphi_i\partial_\nu\varphi_i - \frac{1}{2}\eta_{\mu\nu}\left[(\partial_\kappa\varphi_i)^2 + \mu_i^2(\varphi_i - f_i)^2\right]\right). \tag{36}$$

If we substitute the solution (15) with the radiation field $\varphi^{free} = 0$, we find $\alpha_{(0)}^{sc} = -\alpha_{(2)}^{sc} = -\alpha_{(3)}^{sc}$, or

$$T_{\mu\nu}^{scalar} = \alpha_{(0)}^{sc}\left(n_{(0)\mu}n_{(0)\nu} - n_{(2)\mu}n_{(2)\nu} - n_{(3)\mu}n_{(3)\nu}\right) + \alpha_{(1)}^{sc}\, n_{(1)\mu}n_{(1)\nu}, \tag{37}$$

with

$$\begin{aligned} \alpha_{(0)}^{sc} &= \sum_i \frac{g_i^2}{32\pi^2R^4}\, e^{-2\mu_i R}\left(1 + 2\mu_i R + 2\mu_i^2R^2\right), \\ \alpha_{(1)}^{sc} &= \sum_i \frac{g_i^2}{32\pi^2R^4}\, e^{-2\mu_i R}\,(1 + 2\mu_i R). \end{aligned} \tag{38}$$

Note that, as the eigenvalues $\alpha_{(\lambda)}$ are scalars, they may be evaluated in any reference frame, in particular in the rest frame.

The third contribution comes from the vector fields and is evaluated from

$$T_{\mu\nu}^{vector} = \sum_\alpha \left(F_{\mu\lambda}^\alpha F_\nu^{\alpha\,\lambda} + \mu_\alpha^2 A_\mu^\alpha A_\nu^\alpha - \frac{1}{2}\eta_{\mu\nu}\left[\frac{1}{2}(F_{\kappa\lambda}^\alpha)^2 + \mu_\alpha^2(A_\kappa^\alpha)^2\right]\right). \tag{39}$$

Using the explicit solution (15) with $A_\mu^{\alpha\, free} = 0$ leads to $\alpha_{(0)}^{vec} = \alpha_{(2)}^{vec} = \alpha_{(3)}^{vec}$, hence

$$T_{\mu\nu}^{vector} = \alpha_{(0)}^{vec}\left(n_{(0)\mu}n_{(0)\nu} + n_{(2)\mu}n_{(2)\nu} + n_{(3)\mu}n_{(3)\nu}\right) + \alpha_{(1)}^{vec}\, n_{(1)\mu}n_{(1)\nu}, \tag{40}$$

in which the co-efficients $\alpha_{(0)}^{vec}, \alpha_{(1)}^{vec}$ have the same form as in the case of the scalar fields, up to signs:

$$\begin{aligned} \alpha_{(0)}^{vec} &= \sum_\alpha \frac{q_\alpha^2}{32\pi^2R^4}\, e^{-2\mu_\alpha R}\left(1 + 2\mu_\alpha R + 2\mu_\alpha^2R^2\right), \\ \alpha_{(1)}^{vec} &= -\sum_\alpha \frac{q_\alpha^2}{32\pi^2R^4}\, e^{-2\mu_\alpha R}\,(1 + 2\mu_\alpha R). \end{aligned} \tag{41}$$

Finally we observe, that the constant term $\Lambda\eta_{\mu\nu}$ gives an equal infinite contribution to the stresses and the energy, which only cancels if we take $\Lambda = 0$. Hence we disregard this term from now on. Adding all contributions we can compute the integral

$$\Pi^{\mu\nu} \equiv \int_{\Sigma} d^3x\, T^{\mu\nu}(x) = \sum_{\lambda} \int d^3x\, \alpha_{(\lambda)}\, n^{\mu}_{(\lambda)} n^{\nu}_{(\lambda)}. \tag{42}$$

As explained in eqs.(30)–(32), if the integral is to describe the four-momentum of a real particle, the only non-vanishing contribution to the integral must come from the $\alpha_{(0)}$-component of the stress-energy tensor. All stress components $\alpha_{(i)}$, $i = (1, 2, 3)$ must cancel under the integral. We find that this happens if condition (A) is satisfied: $\sum g_i^2 = \sum q_\alpha^2$, as required to cancel the infinite part of the self-force. Then in the rest frame

$$\Pi^{ij} = \int_{\Sigma} d^3x\, T^{ij} = 0, \tag{43}$$

whilst

$$p^{\mu} = \Pi^{\mu 0} = (Mc, 0, 0, 0), \tag{44}$$

with

$$M = m + \frac{1}{c^2} \sum_i g_i f_i + \frac{1}{8\pi c^2} \left(\sum_i g_i^2 \mu_i - \sum_{\alpha} q_\alpha^2 \mu_\alpha \right). \tag{45}$$

This is the result announced in sect. 1. Because of the way the calculation is organized, by making the Lorentz covariant decomposition (30) of the stress-energy tensor and defining the mass by the frame-independent expression (32), the integral is guaranteed to give a Lorentz covariant result for p_μ.

From expression (45) it follows, that in general the physical mass gets contributions from each of the three possible sources:
1. the mechanical mass m;
2. the vacuum expectation value of the scalar fields f_i;
3. the Coulomb and Yukawa self-energy.

Any of these contributions can vanish for some physical reason, leaving the explanation of the particle mass in unknown mechanics, in scalar vacuum expectation values or in self-energy. Certainly, even if we suppose a purely dynamical (field theoretical) explanation of mass, this does *not* have to reside directly in the vacuum expectation values of the scalar fields; the self-energy terms would suffice in principle. However, it is quite reasonable to expect that the masses of the scalar and vector fields themselves are related to the vacuum expectation values:

$$\mu_i = \sum_j A_{ij} f_j, \qquad \mu_\alpha = \sum_j B_{\alpha j} f_j, \tag{46}$$

where the co-efficients A_{ij} and $B_{\alpha j}$ are functions of the coupling constants between the scalar and vector fields. Then all terms in the equation for the physical mass M become proportional to the vacuum expectation values of the scalar fields.

Of course, the lowest-order (v.e.v.) terms are responsible for generating the full (classical) mass if

$$\sum_i g_i^2 \mu_i - \sum_\alpha q_\alpha^2 \mu_\alpha = 0. \tag{47}$$

Unlike our earlier relations (24), (25), there is no obvious physical need for such a constraint in terms of vanishing self-forces or related conditions. Notice however, that the three constraints (24), (25) and (47) would reduce to a single constraint if the masses of all scalar and vector fields were equal:

$$\mu_i = \mu_\alpha, \qquad \forall (i, \alpha). \tag{48}$$

In the standard model this is certainly not the case at low energies, although it is trivially true in the high-energy limit where all boson masses vanish. But note, that relation (48) is characteristic for supersymmetric theories, especially $N \geq 2$ Yang-Mills models, where the vector and scalar masses are equal as long as supersymmetry is unbroken. Indeed, we can interpret the result (47) as a classical non-renormalization theorem.

5 Hamilton-Jacobi formulation

In this section an alternative derivation of the mass formula (45) is presented, based on the Hamilton-Jacobi formalism. As a starting point, we perform a partial integration in the action (12)

$$\begin{aligned} S_{field} \; = \; & \int d^4x \left\{ - \sum_{i=1}^N \left[\frac{1}{2} \left(\varphi_i - f_i \right) \left(-\Box + \mu_i^2 \right) \left(\varphi_i - f_i \right) \, + \, \rho_i \, \varphi_i \right] \right. \\ & \left. - \sum_{\alpha=1}^M \left[\frac{1}{2} \, A_\mu^\alpha \left((-\Box + \mu_\alpha^2) \eta^{\mu\nu} \, + \, \partial^\mu \partial^\nu \right) A_\nu^\alpha \, - \, \frac{1}{c} \, A_\mu^\alpha \, j_\alpha^\mu \right] \right\} \\ & + \textit{boundary terms}. \end{aligned} \tag{49}$$

In the integrand we substitute the field equations (14), obtaining

$$S_{field}^{cl} = \int d^4x \left\{ - \frac{1}{2} \sum_i \left[\rho_i f_i + \rho_i \varphi_i \right] + \frac{1}{2c} \sum_\alpha j_\alpha^\mu A_\mu^\alpha \right\}. \tag{50}$$

Next we take the explicit solution (15), with the free radiation fields taken to vanish, so as to describe a single non-interacting particle, dressed only with its Coulomb-Yukawa fields, and we use the expressions (13) for the scalar charge and vector current densities. This gives

$$
\begin{aligned}
S^{cl}_{field} &= \int d\lambda \sqrt{-\left(\frac{d\xi^\mu}{d\lambda}\right)^2} \times \\
&\qquad \left[-\sum_i g_i f_i + \sum_i \frac{g_i^2 e^{-\mu_i R}}{8\pi R} - \sum_\alpha \frac{q_\alpha^2 e^{-\mu_\alpha R}}{8\pi R}\right]_{R\to 0} .
\end{aligned} \tag{51}
$$

To obtain the last line, we have substituted for the four-velocity u_μ in the vector potential the expression

$$
u_\mu = \frac{d\xi_\mu}{d\tau} = \frac{1}{\sqrt{-(d\xi^\nu/d\lambda)^2}} \frac{d\xi_\mu}{d\lambda}. \tag{52}
$$

Taking the limit $R \to 0$ and adding the mechanical mass-term to the action finally gives

$$
S^{cl} = S^{cl}_{field} + S^{cl}_0 = -Mc^2 \int d\lambda \sqrt{-\left(\frac{d\xi^\mu}{d\lambda}\right)^2}, \tag{53}
$$

with the total mass M given by expression (45). Note that in order to obtain this result it was *not* necessary to substitute the equation of motion for the particle, except that in equation (51) we have assumed implicitly that the particle moves at constant velocity. Thus we may view this action as an *effective particle action* in the absence of external fields, derived by integrating out the fields from the full Lagrangian.

As one might expect, S^{cl} is precisely of the form of the action for a non-interacting particle, after replacing the mechanical mass m by the full physical mass M. The value of M quoted above was derived on the assumption of constant velocity in the absence of external fields. Therefore, upon substitution of the solution of the equation of motion for a free particle, we obtain Hamilton's principal function

$$
S^{cl}_0 = -Mc\sqrt{-(\xi^\mu_f - \xi^\mu_i)^2} = -Mc^2 \left(\tau_f - \tau_i\right), \tag{54}
$$

from which we derive the expression for the four-momentum

$$
p_\mu(\tau_f) = \frac{\partial S^{cl}_0}{\partial \xi^\mu_f} = M u_\mu. \tag{55}
$$

This is in full agreement with our results from the analysis of the stress-energy tensor.

6 Quantum theory

The models discussed so far are purely classical, and the results obtained may be considered as an extension and completion of the classical electron model of Lorentz and Abraham [10, 11]. In a quantum field-theoretical context, one would expect the results to be only a first approximation, with additional contributions coming from the quantum-polarizability of the vacuum.

As a first step a covariant formalism is required for a quantum field theoretical calculation of the mass which naturally has the result of equation (45) as its first approximation. In quantum field theory, particle masses appear as poles in the propagator. What is needed is a formalism for computing the value of this pole. The approach which is most close in spirit to the classical treatment, and is in fact a direct quantum-extension of the Hamilton-Jacobi procedure, is the path-integral formalism. In this section I describe how to compute various expressions for the propagator in terms of various forms of the classical action. It then becomes clear how to extract the value of the physical mass while taking into account strong-field effects like the contribution from the Coulomb and Yukawa-type fields.

What we have learned from the Hamilton-Jacobi treatment of the interacting particle models is, that the classical action of the full theory for a single particle coupled to scalar and vector fields reduces to that of a free particle, with a (finitely) renormalized value of the mass. We expect the same for the case of quantum theory: the propagator of the interacting theory should behave like that of a free particle, with the pole shifted to a renormalized value of the mass. Therefore it is instructive to study again first the case of a free point particle of mass m, and then proceed to the interacting case. In the classical theory we have used a reparametrization-invariant square-root type of action for the particle, S_0 of eq.(5). An alternative is provided by the quadratic action [12]

$$S_1 \,=\, \frac{m}{2} \int d\lambda \left[\frac{1}{e} \left(\frac{d\xi^\mu}{d\lambda} \right)^2 - c^2 e \right]. \tag{56}$$

which is also reparametrization invariant on account of including the einbein variable $e(\lambda)$. From this the action S_0 can be derived by solving the constraint obtained by varying S_1 with respect to e:

$$c^2 e^2 d\lambda^2 \,=\, -\left(d\xi^\mu \right)^2 \,\equiv\, c^2 d\tau^2, \tag{57}$$

Substitution of the two possible solutions

$$e \,=\, \pm \sqrt{- \left(\frac{1}{c} \frac{d\xi^\mu}{d\lambda} \right)^2}, \tag{58}$$

gives the two actions

$$S_{\pm} \,=\, \mp mc^2 \int d\lambda \sqrt{-\left(\frac{1}{c} \frac{d\xi^{\mu}}{d\lambda} \right)^2} \,=\, \mp S_0. \tag{59}$$

The two solutions are characterized by different directions of the world-line evolution in terms of proper time: $d\tau = \pm e d\lambda$; therefore the actions $S_{\pm}$ can be interpreted as the action of a particle and an anti-particle, respectively [13]. This follows not only from the reversal of the direction of the world-line, but also from the role of the two actions in the quantum theory, as is discussed next.

We begin with the quadratic action S_1, eq.(56), and establish its relation to the Feynman propagator of a free scalar particle:

$$\Delta_F(x-y) \,=\, \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip\cdot(x-y)}}{p^2+m^2-i\varepsilon}, \tag{60}$$

where from now on we take natural units $c = \hbar = 1$. As noted by Schwinger [14], we can write the Feynman propagator as a proper-time integral:

$$\Delta_F(x-y) \,=\, \frac{i}{2m} \int_0^{\infty} d\tau\, K(x-y|\tau), \tag{61}$$

where $K(x-y|\tau)$ is the kernel of the relativistic Schrödinger equation:

$$i \frac{\partial}{\partial \tau} K(x-y|\tau) \,=\, \frac{1}{2m} \left(-\Box_x + m^2 - i\varepsilon \right) K(x-y|\tau), \tag{62}$$

i.e. the solution satisfying the initial condition

$$\lim_{\tau \rightarrow 0} K(x-y|\tau) \,=\, \delta^4(x-y). \tag{63}$$

The explicit expression is

$$K(x-y|\tau) \,=\, -\frac{im^2}{(2\pi\tau)^2}\, e^{\frac{im}{2\tau}(x-y)^2 - \frac{i}{2}(m-i\varepsilon)\tau}. \tag{64}$$

As the kernel satisfies Huygens' principle

$$\int d^4\xi\, K(x-\xi|\tau_1) K(\xi-y|\tau_2) \,=\, K(x-y|\tau_1+\tau_2), \tag{65}$$

a discretized time path-integral is obtained by re-iterating this equation many times:

$$K(x-y|\tau) \,=\, \int \prod_{k=1}^{N} d^4\xi_k \, \prod_{i=0}^{N} K(\xi_{i+1}-\xi_i|\Delta\tau_i), \tag{66}$$

where $\xi_0 = y$, $\xi_{N+1} = x$ and $\sum_i \Delta\tau_i = \tau$. Taking the continuum limit we arrive at a path integral expression for the propagator (cf.[15]):

$$\Delta_F(x-y) \;=\; \frac{i}{2m}\int_0^\infty dT \int_y^x D\xi^\mu(\tau)\, e^{\frac{i}{2}\int_0^T d\tau\left\{m\,\dot{\xi}_\mu^2 - m + i\varepsilon\right\}} \tag{67}$$

The exponent is precisely the quadratic action S_1 after fixing the value of the gauge degree of freedom $e(\lambda) = 1$. This can be done consistently [8], as the corresponding Fadeev-Popov determinant is just a multiplicative constant, which is removed by proper normalization.

Next we consider the Einstein action S_+ and inquire into its meaning in quantum field theory. First we make an observation about its meaning at the classical level. Namely, this action can be considered as describing the motion of the particle in the laboratory frame in which $\xi^0 = ct$ is the time parameter, rather than a dynamical variable. This corresponds to the gauge choice $\lambda = t$, after which the action becomes

$$S_+ \;=\; -m\int dt\,\sqrt{1-\vec{v}^{\,2}}. \tag{68}$$

In this action we can only freely vary the spatial co-ordinates $\vec{x}$. The corresponding phase-space is spanned by these co-ordinates and the momenta

$$\vec{p} \;=\; \frac{m\vec{v}}{\sqrt{1-\vec{v}^{\,2}}}. \tag{69}$$

The time-evolution in the laboratory frame is then described by the Hamiltonian

$$H \;=\; \sqrt{\vec{p}^{\,2}+m^2}. \tag{70}$$

It is straightforward to check that the corresponding Hamilton equations correctly reproduce the laboratory-time dynamics of the relativistic point particle. The Hamiltonian form of the action is

$$S_+ \;=\; \int dt\,\left(\vec{p}\cdot\vec{v} - \sqrt{\vec{p}^{\,2}+m^2}\right). \tag{71}$$

We assert that with $t_1 = y^0$, $t_2 = x^0$, and $\vec{v}(t) = d\vec{\xi}/dt$, the Hamiltonian path integral

$$K^+(x-y) \;=\; \int_{\vec{y}}^{\vec{x}} D\vec{\xi}(t)\int D\vec{p}(t)\, e^{i\int_{t_1}^{t_2} dt\,\left(\vec{p}\cdot\vec{v}-\sqrt{\vec{p}^{\,2}+m^2}\right)}, \tag{72}$$

acquires the meaning of the positive frequency part of the propagator, whilst the action S_- gives the negative frequency part, thereby confirming our earlier interpretation of these actions in the quantum theory.

To prove this assertion, we first note that $K^+(x-y)$ defined above is a solution of the homogeneous Klein-Gordon equation, because

$$i\frac{\partial}{\partial t}K^+(x-y) \;=\; \mathcal{H}_0 K^+(x-y), \tag{73}$$

where $\mathcal{H}_0 = \sqrt{-\Delta + m^2}$. Next we recall the well-known decomposition of the Feynman propagator into positive and negative frequency parts

$$\Delta_F(x-y) \,=\, \theta(x^0 - y^0)\Delta^+(x-y) \,-\, \theta(y^0 - x^0)\Delta^-(x-y), \tag{74}$$

with

$$\Delta^{\pm}(x-y) \,=\, \pm\frac{i}{(2\pi)^3}\int \frac{d^3p}{2\omega_p}\, e^{\pm i\left(\vec{p}\cdot(\vec{x}-\vec{y})-\omega_p(x^0-y^0)\right)}, \tag{75}$$

where as usual $\omega_p = \sqrt{\vec{p}^{\,2} + m^2}$. The positive and negative frequency parts satisfy the inner-product rule

$$\Delta^{\pm}(x-y) \,=\, \int d^3\xi\, \Delta^{\pm}(x-\xi)\, \frac{\overleftrightarrow{\partial}}{\partial \xi^0}\, \Delta^{\pm}(\xi - y). \tag{76}$$

Like Huygens' principle (65) this equation can be reiterated an indefinite number of times, yielding a discretized time expression for a path integral, which in the continuum limit reduces to $K^+(x-y)$ in (72). Thus the path integral constructed from the Einstein action represents a different type of Greens function of the corresponding field theory than the path integral (67) based on the quadratic action.

The generalization of these results to particles interacting with a scalar and a vector field is straightforward. One looks for the kernel of the Schrödinger equation

$$i\frac{\partial}{\partial t}K(x-y|\tau) \,=\, \left(\hat{\mathcal{H}} - i\varepsilon\right)\, K(x-y|\tau), \tag{77}$$

where $\hat{\mathcal{H}}$ is the laplacian operator in the presence of scalar and vector fields:

$$\hat{\mathcal{H}} \,=\, \frac{1}{2m}\left(-(\partial_\mu - qA_\mu)^2 + \frac{g^2}{2}\,\varphi^2\right). \tag{78}$$

$K(x-y|\tau)$ is to satisfy the boundary condition (63) and the Huygens superposition principle (65). The solution of this problem can be written as the path integral

$$K(x-y|T) \,=\, \int D\xi^\mu(\tau)\, e^{\frac{i}{2}\int_0^T d\tau\left\{m\dot{\xi}_\mu^2 - qA\cdot\xi - \frac{g^2}{2m}\varphi^2 - m + i\varepsilon\right\}}. \tag{79}$$

Then the Feynman propagator for a particle in external fields in the interacting theory is again given by eq.(61), with the integrand replaced by the expression (79). Finally, the propagator for such a particle when the fields become dynamical is obtained by functional integration over the scalar and vector fields with a density $\exp(iS_0^{field})$, where S_0^{field} is the kinetic action of the scalar and vector fields.

Now consider the alternative formulation, which may be based upon the Hamiltonian

$$H = \sqrt{(\vec{p} - q\vec{A})^2 + \frac{g^2}{2}\varphi^2} + q\phi, \tag{80}$$

with $\phi = A^0$. This Hamiltonian gives the same classical equations of motion as the action in the exponent in (79). However, it is a Hamiltonian for time-evolution in the laboratory frame, rather than proper time, and the corresponding path integral

$$K^+(x-y) = \int_{\vec{y}}^{\vec{x}} D\vec{\xi}(t) \int D\vec{p}(t)\, e^{i\int_{t_1}^{t_2} dt\left(\vec{p}\cdot\vec{v} - H(\vec{p},\vec{\xi})\right)}, \tag{81}$$

is a solution of the homogeneous Klein-Gordon equation

$$\left(\frac{\partial}{\partial t} - q\phi\right)^2 K^+(x-y) = \left[\left(\vec{\nabla} - q\vec{A}\right)^2 - \frac{g^2}{2}\varphi^2\right] K^+(x-y). \tag{82}$$

With dynamical scalar and vector fields, one should again perform a functional integral over the fields with the weight $\exp(iS_0^{field})$. The interesting observation, following from the classical Hamilton-Jacobi formalism presented above, is that by expanding the fields and the particle paths about the correct classical solutions (15) and (20), modulo higher order quantum corrections one finds that the Green's functions Δ_F and $\Delta^{\pm}$ in the interacting theory still satisfy the decomposition (74), provided one replaces the free mass m everywhere by the finite physical mass M of eq.(45). Thus to this approximation the light-cone structure of the theory, implying causality, and the invariant distinction between particles and anti-particles is preserved in the interacting quantum theory. However, further calculations to investigate higher order quantum corrections (loops) remain to be done.

7 Discussion

In this paper I have presented a consistent theory of classical point charges. The model is interesting in itself, because it shows how particle masses become computable in terms of field parameters (coupling constants, vacuum expectation values and characteristic ranges) once the particle is intrinsically stable.

At first sight, the stability criterion seems to have little relevance for particle physics phenomenology, even at tree level; however, such a comparison may be premature. First of all, we have chosen to analyse here the simplest model with only abelian couplings, because of the advantage that it can be solved completely. Secondly, nothing definite can be said about the scalar sector of the standard model: the number of scalar fields (e.g. Higgs doublets) remains unknown, and

their Yukawa couplings are completely arbitrary (as are the masses of quarks and leptons). Also, new heavy gauge bosons could enter into the stability relations (24), (25). Furthermore, the effects of spin have been ignored. It seems likely that adding fermions to the model could further improve its behaviour, for example by the interplay with one or more supersymmetries.

In addition, in realistic applications one has to take into account quantum effects, related to the many-body nature of quantum field theory: pair creation, (anti-)screening and renormalization. In general, the contributions of these effects to masses and couplings as computed in perturbation theory are divergent; this renders the classical value of the mass meaningless. Also, it is often argued that since only the total (effective) mass is observable, the contribution of scalars, vectors and vacuum expectation values cannot be separated and the notion of Coulomb- and Yukawa-energy contributing to the inertia of the particle has no operational meaning.

Commenting first on the latter argument, it is clear that if the vacuum expectation value and range of scalar and vector fields can change, as during phase transitions, then the relative contributions of fields to the stability conditions and to the mass vary and certainly the changes in these quantities are observable. At least in theory, therefore, the various contributions to the mass do seem to be physically distinguishable. Our results then imply constraints on the changes in the values of the field parameters during phase transitions.

As concerns the contribution of quantum effects to the mass, there is no a priori reason why it should *not* be computable, like the classical mass. In fact, the BPS solutions [16, 17] in supersymmetric field theories are believed to provide examples of this. This is significant, because the stability of classical monopole solutions is also guaranteed precisely because of the interplay between vector and scalar fields [18]-[20]. More generally, the ultra-violet divergences one encounters in perturbation theory are the result of short distance fields which cannot be controlled even if the coupling constant is arbitrarily small: for *any* non-zero value of (g, q) the classical Yukawa/Coulomb field becomes large as soon as the distance approaches $R \approx g\lambda_C/4\pi$, where $\lambda_C = \hbar/Mc$ is the Compton wavelength of the particle. Therefore, when computing the effect of quantum fluctuations on the one-particle state it is obviously important to expand the fields around the correct classical solution, which includes the large short-distance Coulomb and Yukawa fields, and not about the vacuum state. Indeed, there is no reason to think that a naive expansion in weak fields close to the vacuum would be a good approximation to the quantum corrections at all, except for the large-distance part.

Of course, even when taking into account the singular part of the fields in an improved perturbation theory, finiteness of the result for the mass is not necessarily guaranteed. In particular, there is an interplay with other effects, like coupling constant renormalization. But the remarkable properties of supersymmetric gauge theories involving scalars ($N \geq 2$ in four dimensions) may be an

indication of the viability of the scheme. The necessary calculations certainly involve interesting physical and computational problems.

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